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Effect of an Adjustable Admittance on the Macroscopic Energy Levels of a Current Biased Josephson Junction

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Abstract

We have measured the rate of macroscopic quantum tunneling out of the zero voltage state of a current biased Josephson junction shunted by a microwave circuit whose resonant frequencies were varied in situ. By irradiating the junction with microwaves, we find that when the transition frequency between the junction levels coincides with a resonant frequency of the circuit, an hybridization of the junction and circuit quantum states occurs.

1. Introduction

Electronic circuits based on semiconductor devices always behave classically in the sense that currents flowing in them are macroscopic variables obeying classical equations. The words "quantum electronics" only mean that the devices are based on the microscopic quantum behavior of individual electrons. In a tunnel diode [1] for example, the negative slope of the voltage to current relationship originates in the quantum mechanical tunneling of the carriers across the junction. However, at the macroscopic level, the switching properties of the diode when it is coupled to an adequate bias circuit can be explained in purely classical terms once the voltage to current relationship is known.

In recent years, by contrast, experiments [2–4] involving Josephson junctions at dilution refrigerator temperatures have demonstrated that a macroscopic electrical variable can behave quantum-mechanically. In the case of the current-biased junction, the macroscopic variable is the phase difference δ between the superconducting order parameters on either side of the barrier. The observations show that the junction, albeit its underlying complexity, can be thought of as a kind of one dimensional atom in an electrical field, the phase difference δ playing the role of the electron position r. The junction is equivalent to the system consisting of a particle of coordinate δ and mass $C(\Phi_0/2\pi)^2$ moving in a potential [5] $U(\delta) = -I_0\Phi_0/2\pi[\cos\delta + (I/I_0)\delta]$. Here I_0 and C are the junction critical current and capacitance, I is the bias current and Φ_0 the flux quantum $h/2e$. The voltage across the junction is proportional to the velocity of the particle. The zero-voltage state of the junction corresponds to the confinement of the particle of coordinate δ to one well of the tilted cosine potential and is analogous to the bound state of the electron in the Coulomb potential of the nucleus. When the particle escapes from this metastable state it runs freely down the potential and a voltage appears across the junction. The detection of the switching to this voltage state is analogous to the detection of the ionization of the atom.

In the actual conditions of the experiments, the reduced bias current $x = I/I_0$ is very close to one and the potential from which the particle escapes is, to a very good approximation, a cubic potential with barrier height $\Delta U = U_0(1-x)^{3/2}$ and oscillation frequency at the bottom of the well $\omega_p = (L_{\mathrm{eff}}C)^{-1/2}$, where $U_0 = 2\sqrt{2}I_0\Phi_0/3\pi$ and $L_{\mathrm{eff}} = [(\Phi_0/2\pi I_0)(1-x^2)]^{1/2}$ (see Fig. 1). A previous experiment has demonstrated the existence of quantized energy levels [4] by measuring the enhancement of the escape rate in presence of a microwave irradiation that induces transitions between levels. Theoretical calculations of energy level positions and widths that treat perturbatively the coupling of the junction with its shunting circuit are in quantitative agreement with the experimental results [6, 7].

These calculations are related to a central problem in atomic physics: the modification of the energy levels of an atom induced by its coupling with the radiation field. For the current biased junction, the role of the field is played by the microwave current in the circuit shunting the junction. In the aforementioned experiment this circuit behaved as a capacitor, which simply renormalized the junction capacitor, in parallel with a resistor. In atomic physics this resistor corresponds to the free vacuum.

A richer situation is the case of the confined field which has quasi-discrete modes. This case has recently been adressed experimentally in the investigation of the shifts of the energy levels of an atom placed in an optical resonator [8]. We have built the quantum network [9] analog of that experiment: we couple a Josephson junction to a microwave resonating circuit that can be varied in situ. We report in this paper the observation of the modulation of the shift and width of the junction energy levels induced by the variation of the resonating mode frequency of the circuit.

2. Experimental apparatus and procedures

A $10 \times 10\,\mu$m Nb–NbO$_x$–PbIn cross-strip tunnel junction was patterned on a Si wafer using standard sputtering, photolithography and evaporation techniques. The junction chip, 3×3 mm, was then mounted at the end of a coplanar transmission line constructed from a fiberglass-copper printed circuit board. Electrical connections between the line and the junction were made with indium pads. A microwave absorbing block, referred to in the following as "the load", was pressed on the coplanar line at a distance L from the indium pads. It defines a section of free line loaded by a section of lossy line of lower characteristic impedance.

Measurements at microwave frequencies with a network analyzer indicate that viewed from the junction, the coplanar line with the load can be modelled as an ideal transmission

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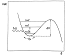

Fig. 1. Cubic potential involved in the motion of the particle representing the junction in its zero voltage state when it is biased slightly below its critical current. Transitions between quantized energy levels in the well can be induced by applying a microwave excitation at the transition frequencies.

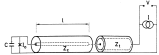

Fig. 2. Microwave circuit whose quantum behavior is tested in the experiment. It consists of a Josephson junction (cross) of critical current I_0 shunted by its self-capacitance C and by a transmission line of characteristic impedance Z_c followed by an attenuating transmission line of characteristic impedance Z_1. The length of the first line is l and its wave propagation velocity v. The junction is d.c. biased by a current source I using the inner and outer conductors of the transmission lines. The d.c. voltage V across the junction is measured using the same conductors.

line of characteristic impedance $Z_c = 75 \pm 5\,\Omega$ and length $l = L + D$ terminated by a resistance $Z_1 = 15 \pm 3\,\Omega$ (see Fig. 2). The dead length D, estimated to be $D = 6$ mm, accounts for the length of the strips on the junction chip and for the distortions of the electromagnetic fields due to the impedance discontinuities at the indium pads and at the load. The velocity of electromagnetic waves along the transmission line is $v = 1.8 \pm 0.1 \times 10^8$ m/s. The assembly was enclosed in a radiation tight copper enclosure and thermally anchored to the mixing chamber of a dilution refrigerator. The load position L was adjusted by means of a string which was pulled from the top of the cryostat and was monitored *in-situ* with a potentiometric position sensor. The junction was d.c. current biased using the inner and outer conductors of the coplanar line. An insulating coating of the microwave absorbing block ensured that the junction, while being loaded at microwave frequencies, stayed unshunted for d.c. currents. In addition to the d.c. bias, the junction was submitted to a microwave current source that consisted of an open-ended coaxial cable brought within 2 mm of the junction and connected through the copper shield to a microwave generator. Electrical connections to room temperature electronics were made through a series of filters that prevented thermal and parasitic noise from reaching the junction and which have been described previously [4].

The temperature of the copper enclosure was determined using a combination of a calibrated germanium thermometer, a ^{60}Co orientation thermometer and a series of NBS fixed points [10]. The lowest measured temperature is 16 mK. The accuracy of the temperature scale and the absence of a significant level of external noise in the copper shield was checked by using a second junction with a critical current and capacitance such that it stayed in the thermal regime even at the lowest temperature.

We measured the escape rate of the junction out of the zero-voltage state as a function of the bias current, with and without microwave irradiation, using the technique of Fulton and Dunkelberger [11]. The microwave power P was adjusted to give a relative increase $\varrho = [\Gamma(P) - \Gamma(0)]/\Gamma(0)$ of the escape rate Γ such that $\max(\varrho) = 2$; values of $\Gamma(0)$ were in the range 10^2 to $10^4\,\mathrm{s}^{-1}$. We collected typically 3×10^4 switching events. After the junction had switched to the voltage state, the bias current was turned off within 30 µs. We ensured that the elapsed time before the next switching event was sufficient for any rise in temperature that occurred in the dissipative state to have become negligible. We changed the position of the load by steps of 1 or 2 mm. At each step, the heat generated by friction of the block on the coplanar line raised the temperature of the mixing chamber to about 50 mK and we had to wait for one hour before the data collection could be resumed at the lowest temperature.

3. Results and discussion

Escape rates measurements performed in the classical regime as described in reference [4] were in good agreement with those predicted theoretically [12]. These measurements allowed a determination of $I_0 = 24\,160 \pm 10$ nA and $C = 11.5 \pm 0.5$ pF. During these measurements the position of the load was kept at its minimal value $L = 0$ mm for which the model of the junction shunted by a smoothly frequency dependent admittance is valid. When L was varied we observed the oscillations of the thermal escape rate predicted by Grabert and Linkwitz [13]. These observations provide an independent determination of the dead length D as well as the plasma frequency ω_p and will be described in a forthcoming publication [14].

At temperatures below 20 mK, only the ground state in the well is populated and the escape rate in the absence of microwaves became temperature independent, with a value in good agreement with predictions for macroscopic quantum tunneling [15]. In the presence of microwaves one expects the escape rate to be resonantly enhanced over its value in the absence of microwaves when the microwave photon energy matches the spacing between the ground state and the first excited state. Note that the tunneling rate out of the excited state is three orders of magnitude greater than out of the ground state so that a relatively small population of the excited state leads to a large increase in the escape rate. We detect this resonance by varying the energy level spacing with the bias current while keeping the microwave frequency fixed. Figure 3 illustrates the results for $L = 10$ mm and a microwave frequency of 3.6 GHz. We define the resonance current I_{RES} as the current corresponding to the maximum of the resonance curve. In Fig. 4 we plot I_{RES} as a function of the load position L for three microwave frequencies. We observe that, at constant microwave irradiation frequency, as the length is increased the resonant current is first shifted towards

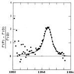

Fig. 3. Microwave induced enhancement of the escape rate as a function of the bias current at $T = 17$ mK. The irradiation frequency was $\Omega = 3.6$ GHz and the load was positioned at $L = 10$ mm. Solid line is a guide for the eye. The maximum of the resonance defines the current I_{RES}. Statistical scatter at the left and right of the resonance curve is typical of results obtained using the technique of Ref. [11].

lower values. A broadening of the resonance curve accompanies this decreasing trend. Then, when the length corresponding to a half wavelength at the irradiation frequency is reached, I_{res} makes a sudden swing to higher values and then resumes its downward trend. The same behavior is repeated at the length corresponding to one wavelength at the irradiation frequency. When the swing occurs, the line becomes so broad that the maximum of the resonance becomes undetectable. This is why, for each frequency, I_{RES} is not plotted at certain positions.

We can get some insight on this behavior in the following way: when the plasma frequency ω_p is not too close to the natural resonance frequencies of the line, the coupling of the junction to the line can be thought of as modifying the effective capacitance of the junction $C \to C + \omega_p^{-1}\,\mathrm{Im}[Y(\omega_p)]$ where $Y(\omega)$ is the admittance of the line viewed by the junction at frequency ω. Here we neglect the difference of order 10% between ω_p and the transition frequency ω_{01} between the ground state and the first excited state. As the length is increased the imaginary part of the microwave admittance at the plasma frequency is modulated in a periodic manner, the period being the half wavelength at the plasma frequency $\pi v/\omega_p$. Consequently I_{RES}, which is essentially the current at which the effective inductor will resonate with the effective capacitor at the irradiation frequency, will vary periodically with the position of the load.

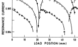

Fig. 4. Resonant current as a function of the position of the load for three irradiation frequencies. Crosses: $\Omega = 3.8$ GHz; open circles: $\Omega = 3.6$ GHz; full circles: $\Omega = 3.4$ GHz. Solid lines are guide for the eye. The half-wavelengths at the three frequencies are respectively 30, 32 and 34 mm.

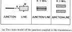

Fig. 5. (a) Two state model of the junction coupled to the transmission line when the energy of the first excited state coincides with the energy of a one photon state of the line. K is the coupling strength and W^{-1} is the lifetime of the photon state. (b) Eigenstates of the junction + line system in the weak coupling regime: a perturbative treatment of the influence of the line on the junction is applicable. (c) Eigenstates of the junction + line system in the strong coupling regime; the junction and line states have lost their identity and have formed two hybridized states.

However, this simple model cannot explain an interesting feature of the data presented in Fig. 4. At positions in the vicinity of the wavelength at the plasma frequency *two* resonances corresponding to two distinct irradiation frequencies can be found at the same current, i.e., the curves $I_{RES}(L, \Omega)$ cross one another. This means that at a fixed current, there is no longer only one transition between the junction ground state and the first excited state but two transitions to a split excited state. This can be explained by considering the junction and the line not as separate entities but forming a global system junction + line (see Fig. 5). At $I = 2\pi v/\omega_p(I)$, if one ignores the coupling between the junction and the line, the first excited state of the junction coincides with the one-photon state of the one-node mode of the line. This photon state has a finite width W due to the non zero termination resistance and is coupled to the junction state with strength K. In an analysis that ignores the coupling with other states two cases must be considered: (1) $K < W/4$, perturbation theory is valid and the junction state simply acquires a width $w = W/2 - 2[(W/4)^2 - K^2]^{1/2}$. (2) $K > W/4$, perturbation theory is no longer valid and the levels loose their identity. One finds instead two levels of equal width $w = W/2$ separated by $\Delta\omega = [K^2 - (W/4)^2]^{1/2}$. The condition for the breakdown of perturbation theory can be expressed in terms of the electrical parameters of the junction and its shunting circuit. One finds that breakdown occurs at $I = 2\pi v/\omega_p(I)$ if $(2\pi Z_j/Z_c)^{1/2} > |\log[(Z_c - Z_t)/(Z_c + Z_t)]|/4$ where $Z_j = (C\omega_p)^{-1}$. This condition is satisfied for the parameters of our experiment.

4. Conclusion

Our results for the modification of the energy levels of the zero voltage state of a current biased Josephson junction induced by the coupling with a transmission line show that perturbation theory cannot account for all the features of the data. A perturbative analysis gives a qualitative understanding of the shifts and widths of the levels when the junction transition frequency is not too close to the resonant frequencies of the line. However when these frequencies match, an "hybridization" of the states of the junction and the line occurs. These results constitute evidence that the multicomponent electronic network composed of the junction and its microwave shunting circuit has to be treated as a whole quantum-mechanical system.

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